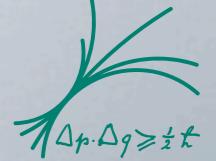




MAX-PLANCK-GESELLSCHAFT

# Automated one-loop calculations with GoSam 2.0



Max-Planck-Institut für Physik  
(Werner-Heisenberg-Institut)

**Gudrun Heinrich**

*Max Planck Institute for Physics, Munich*

In collaboration with

G.Cullen, H.van Deurzen, N.Greiner, G.Luisoni, P. Mastrolia, E. Mirabella, G. Ossola,  
T. Peraro, J. Reichel, J. Schlenk, J.F. von Soden-Fraunhofen, F. Tramontano

**LoopFest 2014**

New York City College of Technology

# Particle physics after the Higgs discovery

- **the big question:** *is there something beyond the clouds (SM) ?*
- *how to find out in the absence of “smoking gun” signals ?*



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  - higher order corrections (QCD, EW)
  - N(N)LO + parton shower matching
  - quark mass effects
  - reduction of PDF uncertainties
  - ...



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# NLO automation

- already entered “phase 2”:
- moved from “proof of concept” multi-particle one-loop calculations towards **validated automated tools** with direct link to phenomenological analysis/experiment
- NLO **matched to parton shower** is new state of the art

many automated NLO tools, e.g.

FeynArts/FormCalc, Grace, BlackHat,  
Helac-NLO, aMC@NLO, NJet, OpenLoops,  
Recola, VBFNLO, MCFM, ... , **GoSam**



# GoSam-2.0

arXiv:1404.7096

program available at

<http://gosam.hepforge.org>

very simple usage

example input file for

$$e^+ e^- \rightarrow t \bar{t}$$

```
process_path=eett
in=    e+, e-
out=   t, t~
order= gs, 0, 2
```



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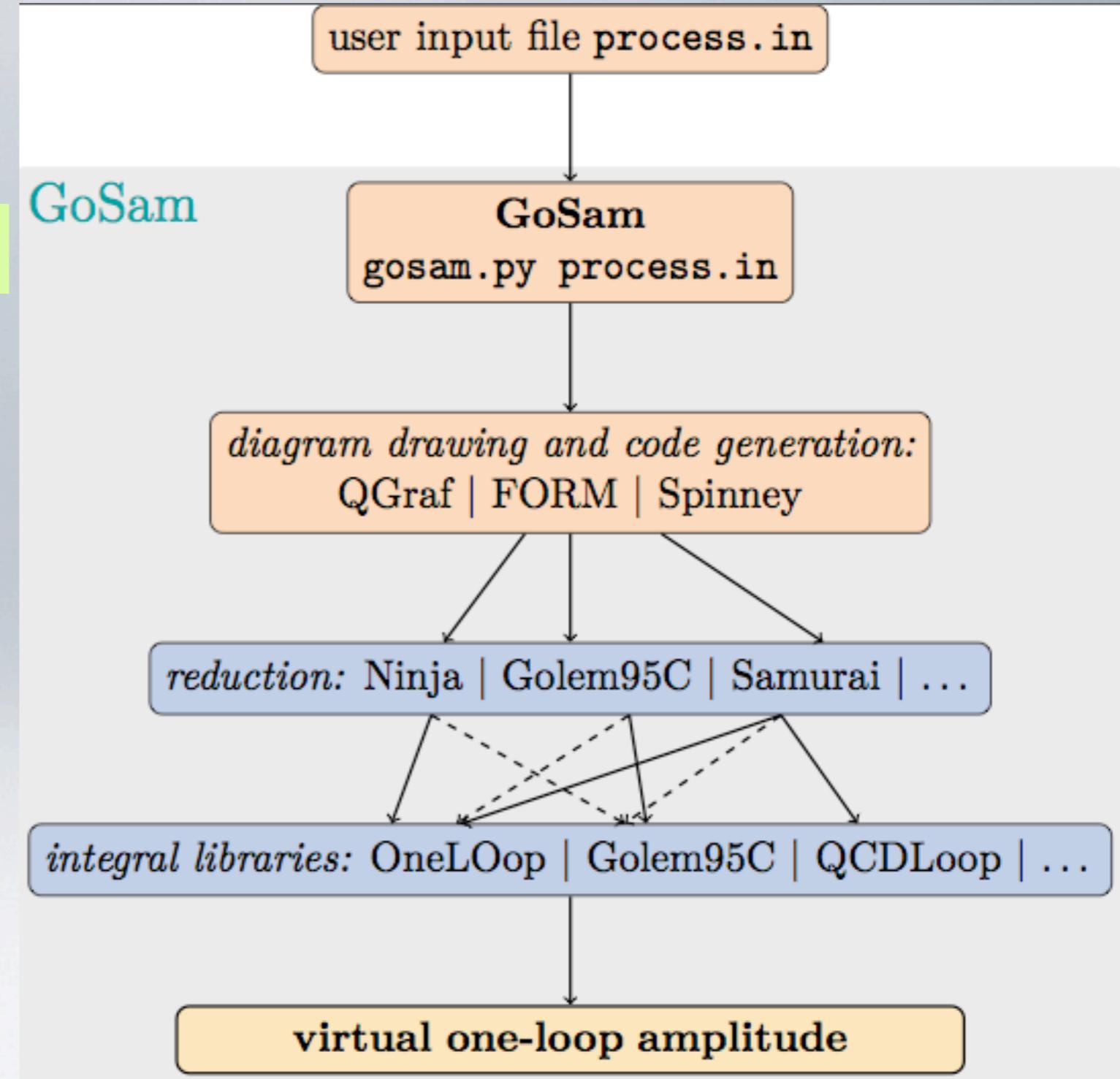
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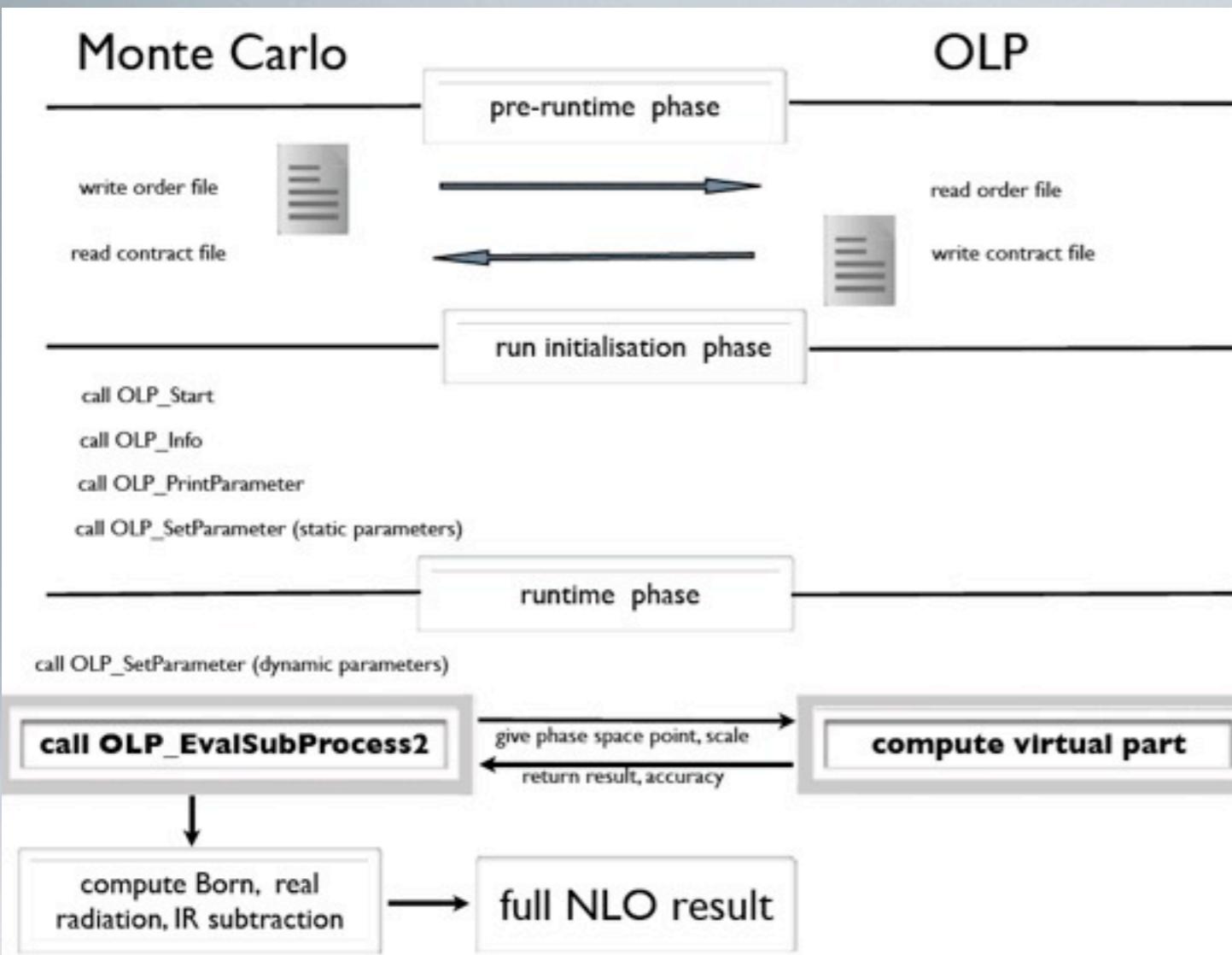
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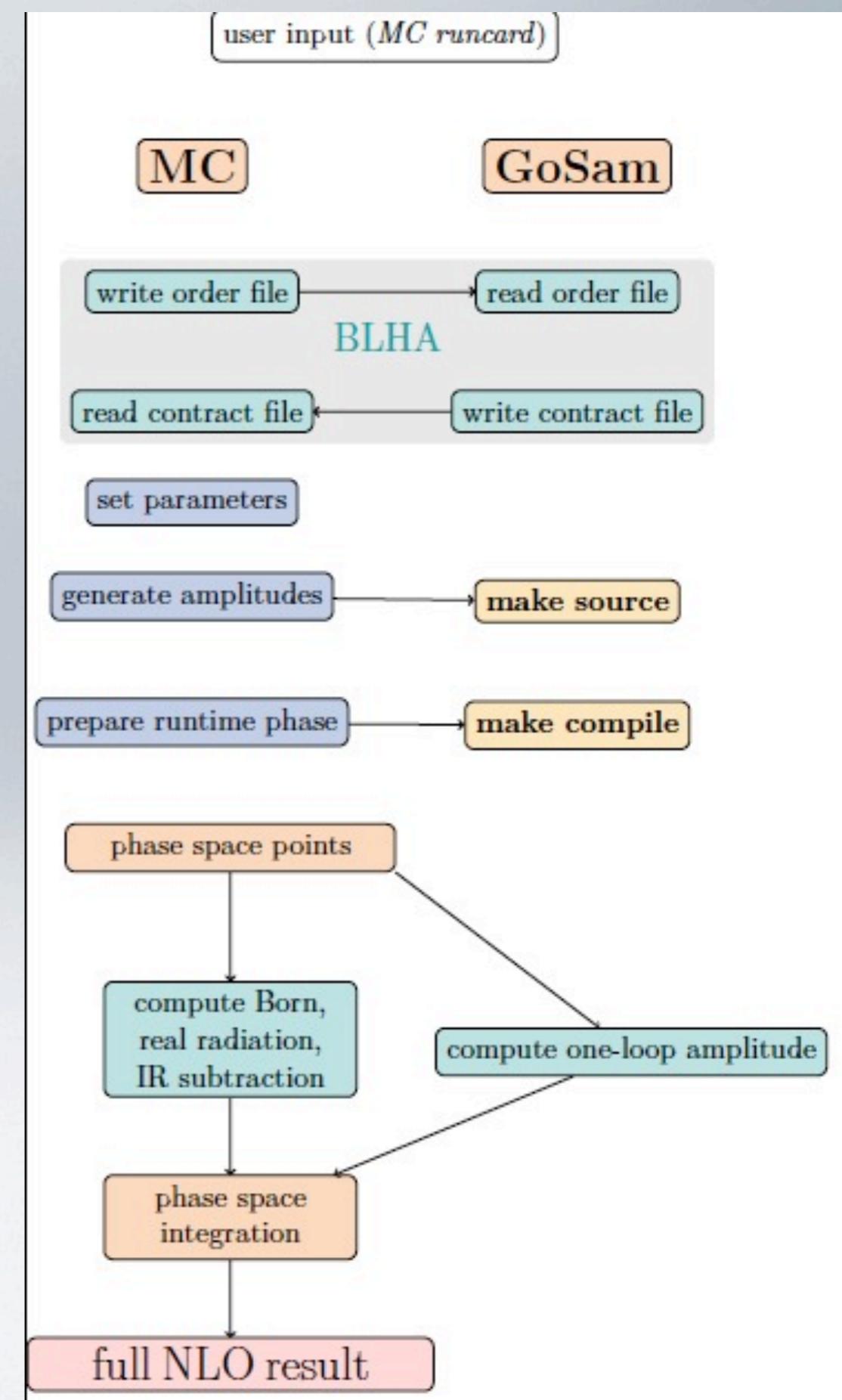


# Interface to Monte Carlo programs

both original Binoth-Les-Houches-Accord  
and extended standards [CPC 185 (2014)]  
are supported

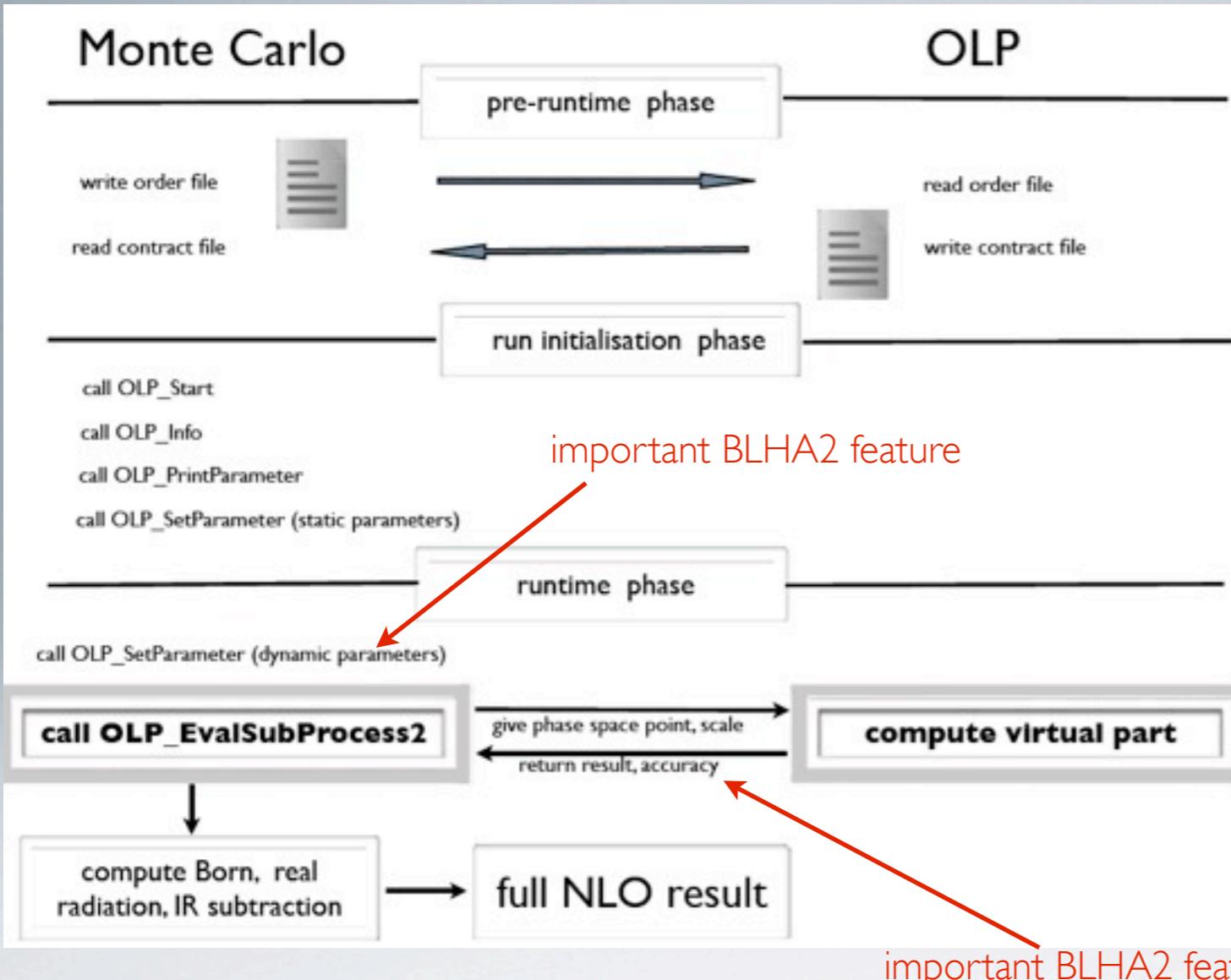


allows combination with  
different MC programs

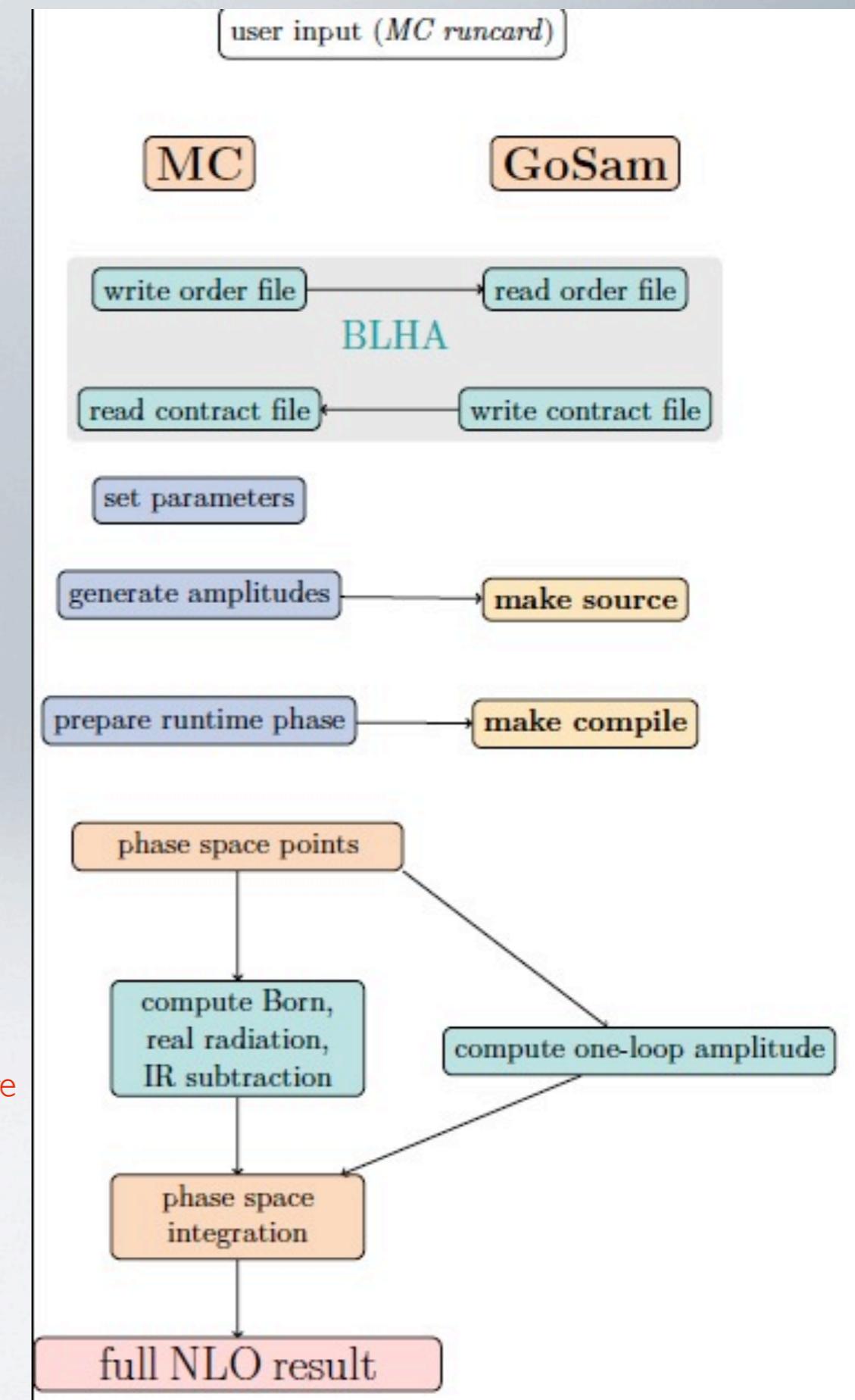


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# Examples of processes calculated with GoSam

- GoSam + MadDipole/MadGraph/MadEvent

$pp \rightarrow W^+W^- + 2\,jets$	[Greiner, GH, Mastrolia, Ossola, Reiter, Tramontano '12]
$pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 + jet$	[Cullen, Greiner, GH '12]
$pp \rightarrow (G \rightarrow \gamma\gamma) + 1\,jet$	[Greiner, GH, Reichel, von Soden-Fraunhofen '13]
$pp \rightarrow \gamma\gamma + 1, 2\,jets$	[Gehrman, Greiner, GH '13]
$pp \rightarrow HH + 2\,jets$	[Dolan, Englert, Greiner, Spannowsky '13]

- GoSam + Sherpa

$pp \rightarrow W^+W^+ + 2\,jets$	[Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano '12]
$pp \rightarrow H + 2\,jets$	[van Deurzen, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, von Soden-Fraunhofen, Tramontano '13]
$pp \rightarrow W^+W^- b\bar{b}$	[GH, Maier, Nisius, Schlenk, Winter '13]
$pp \rightarrow t\bar{t} + 0, 1\,jet$ (includes shower)	[Höche, Huang, Luisoni, Schönherr, Winter '13]
$pp \rightarrow H t\bar{t} + 0, 1\,jet$	[van Deurzen, Luisoni, Mastrolia, Mirabella, Ossola, Peraro '13]

- GoSam + Powheg (includes shower)

$pp \rightarrow HW/HZ + 0, 1\,jet$	[Luisoni, Nason, Oleari, Tramontano '13]
------------------------------------	--

- GoSam + Herwig++/Matchbox (includes shower)

$pp \rightarrow Z + jet$	[Bellm, Gieseke, Greiner, GH, Plätzer, Reuschle, von Soden-Fraunhofen '13]
--------------------------	--

- GoSam + MadDipole/MadGraph/MadEvent + Sherpa

$pp \rightarrow H + 3\,jets$	[Cullen, van Deurzen, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano '13]
------------------------------	---

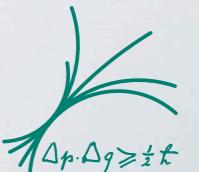
# New features of GoSam 2.0

- Improvements in code generation  
more compact code, faster evaluation



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- Extended range of applicability  
EW schemes, complex masses, effective vertices, higher tensor ranks, BSM physics



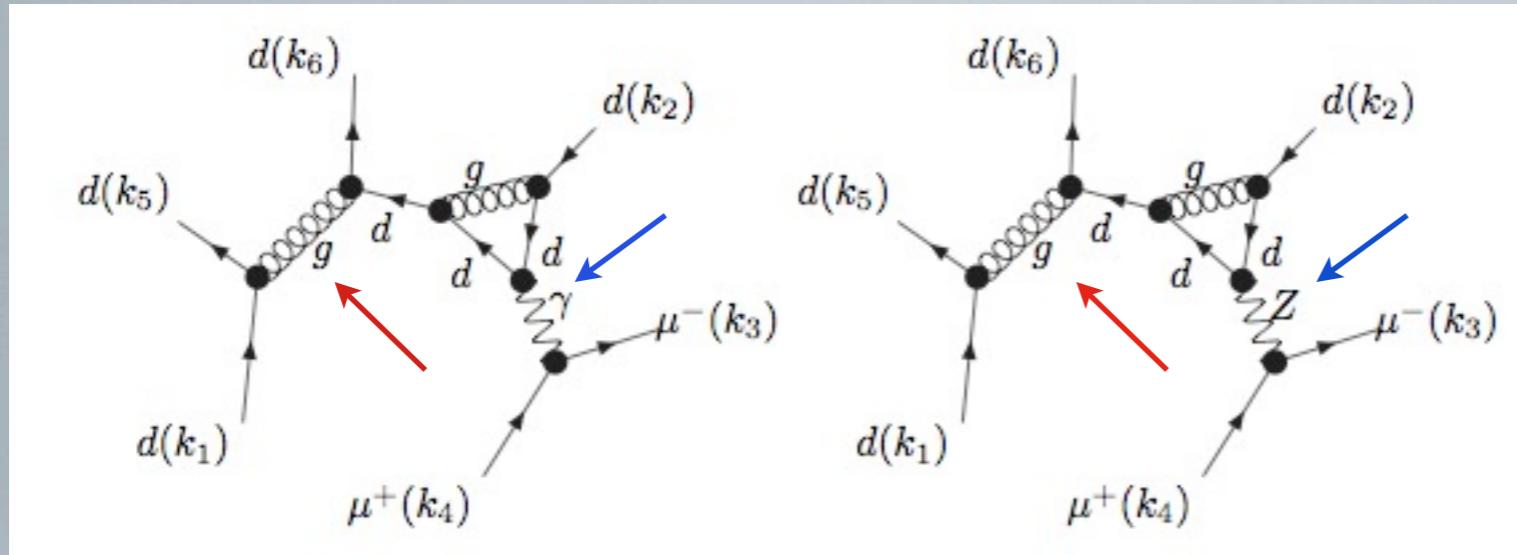
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- Easy installation  
installation script installs and builds the code and all libraries

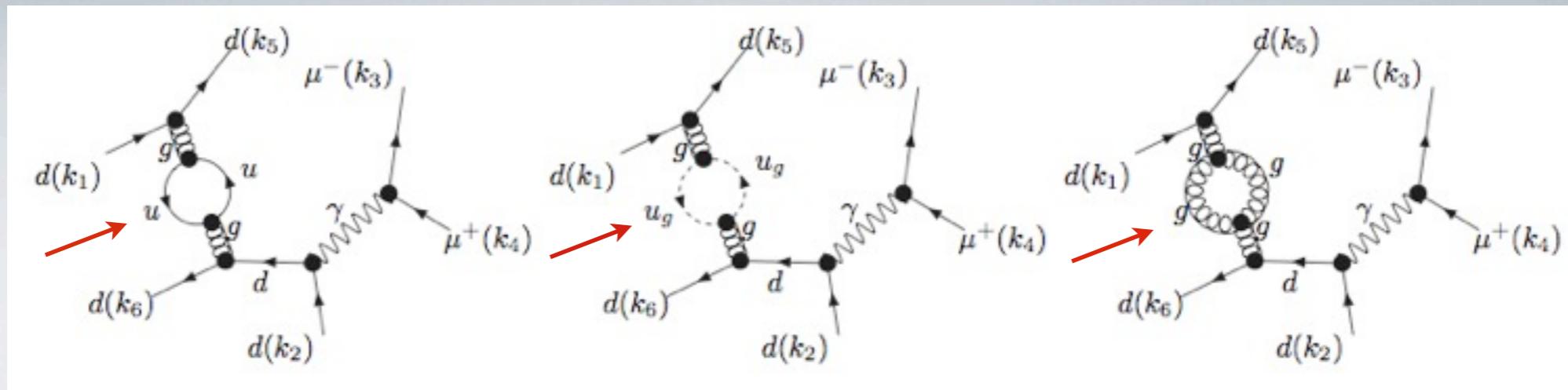


# New code generation methods

- code optimisation with FORM version 4 [Vermaseren, Kuipers, Ueda, Vollinga ]
- construction of “meta-diagrams” from diagrams sharing common substructures



share a tree sub-diagram



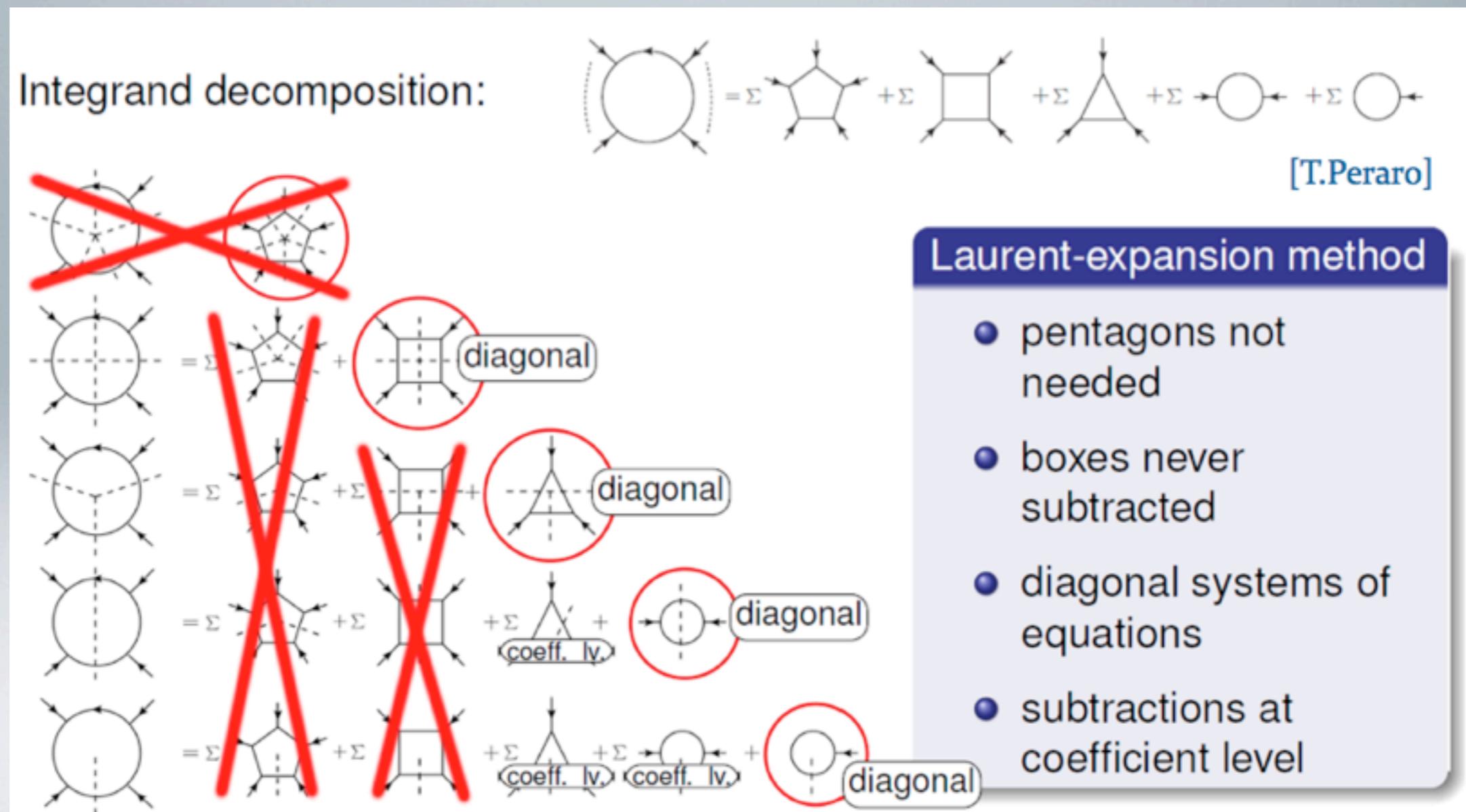
share a loop sub-diagram



# New reduction methods

basic idea: extract the coefficients of the residues of a loop integral by performing a Laurent expansion of the integrand [Mastrolia, Mirabella, Peraro '12]

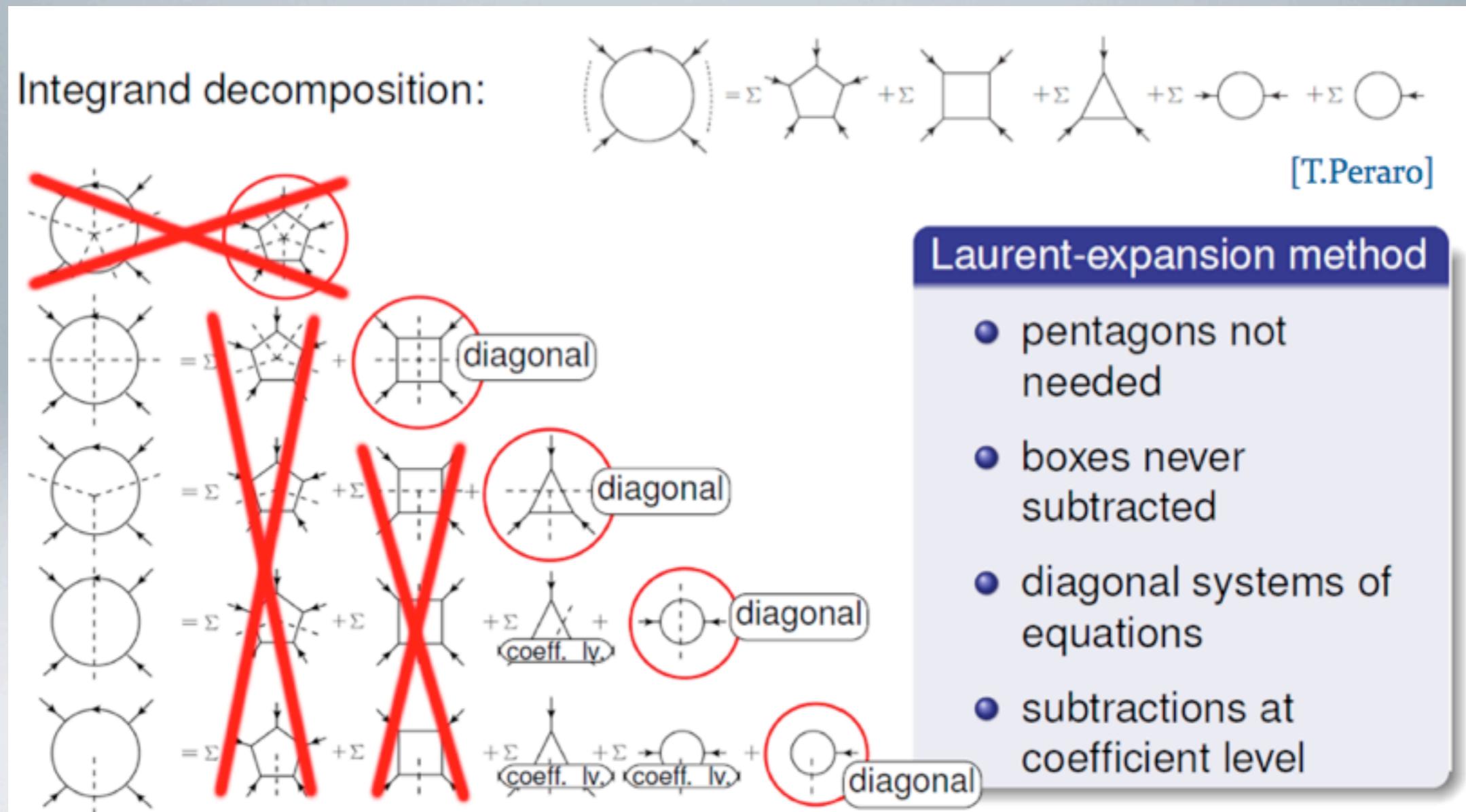
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implemented in the code **Ninja** [T. Peraro '14] see talk of Tiziano Peraro this afternoon



# Reduction methods

- in GoSam-2.0 several reduction libraries available:
  - **Ninja** [van Deurzen, Luisoni, Mastrolia, Mirabella, Ossola, Peraro '13, Peraro '14]  
integrand reduction
  - **Golem95C** [Binoth, Cullen, Guillet, GH, Pilon, Reiter et al. '08, '11]  
tensor reduction (+tensorial reconstruction) [GH, Ossola, Reiter, Tramontano '10]
  - **Samurai** [Mastrolia, Ossola, Reiter, Tramontano '10]  
integrand reduction
- switch between different reduction algorithms “on the fly”  
⇒ flexible rescue system for problematic points  
use tensor reduction when integrand reduction does not pass stability test

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**new:**

all reduction programs,  
**Ninja, Golem95C, Samurai**  
have been extended to support  
higher rank integrals

**Ninja, Samurai:**

van Deurzen, Mastrolia, Mirabella, Ossola,  
Peraro '13, '14

**Golem95C:**

Guillet, GH, von Soden-Fraunhofen '13

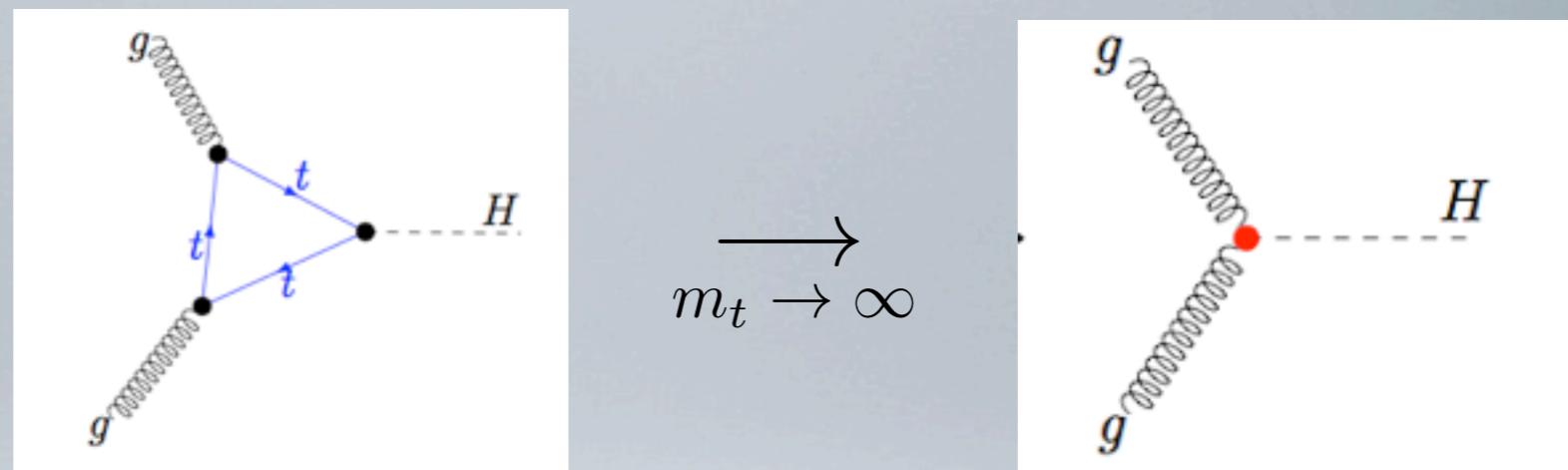
# higher rank tensor integrals

$$I_{\textcolor{red}{N}}^{n,\mu_1 \dots \mu_r}(S) = \int d^n k \frac{k^{\mu_1} \dots k^{\mu_r}}{\prod_{i=1}^N ((k + r_i)^2 - m_i^2 + i\delta)}$$

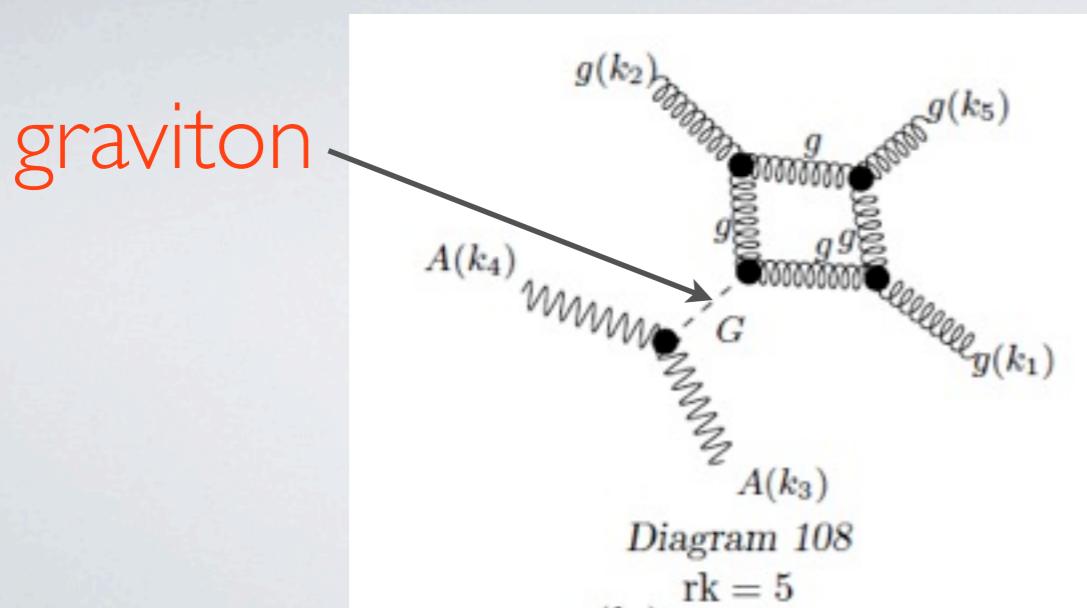
with  $r \geq N + 1$

- needed for example in

- effective theories



- BSM models involving spin-2 particles



rank five box integral due to  
graviton-g-g coupling

$$\begin{aligned} & \frac{-i\kappa\delta_{ab}}{2} \left( p_2^\rho p_1^\sigma g^{\mu\nu} + p_1^\rho p_2^\sigma g^{\mu\nu} - p_1^\nu p_2^\sigma g^{\mu\rho} - p_1^\nu p_2^\rho g^{\mu\sigma} \right. \\ & \quad \left. - p_2^\mu p_1^\sigma g^{\nu\rho} + p_1 \cdot p_2 g^{\mu\sigma} g^{\nu\rho} - p_2^\mu p_1^\rho g^{\nu\sigma} \right. \\ & \quad \left. + p_1 \cdot p_2 g^{\mu\rho} g^{\nu\sigma} + p_2^\mu p_1^\nu g^{\rho\sigma} - p_1 \cdot p_2 g^{\mu\nu} g^{\rho\sigma} \right) \end{aligned}$$

# new range of applicability

- electroweak scheme choice

ewchoice	input parameters	derived parameters
1	$G_F, m_W, m_Z$	$e, \sin \theta_w$
2	$\alpha, m_W, m_Z$	$e, \sin \theta_w$
3	$\alpha, \sin \theta_w, m_Z$	$e, m_W$
4	$\alpha, \sin \theta_w, G_F$	$e, m_W$
5	$\alpha, G_F, m_Z$	$e, m_W, \sin \theta_w$
6	$e, m_W, m_Z$	$\sin \theta_w$
7	$e, \sin \theta_w, m_Z$	$m_W$
8	$e, \sin \theta_w, G_F$	$m_W, m_Z$

# new range of applicability

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- support of complex masses

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8	$e, \sin \theta_w, G_F$	$m_W, m_Z$

complex masses/parameters in generated code and in loop integrals supported

$$m_V^2 \rightarrow \mu_V^2 = m_V^2 - i m_V \Gamma_V, \quad V = W, Z$$

$$\cos^2 \theta_W = \mu_W^2 / \mu_Z^2$$

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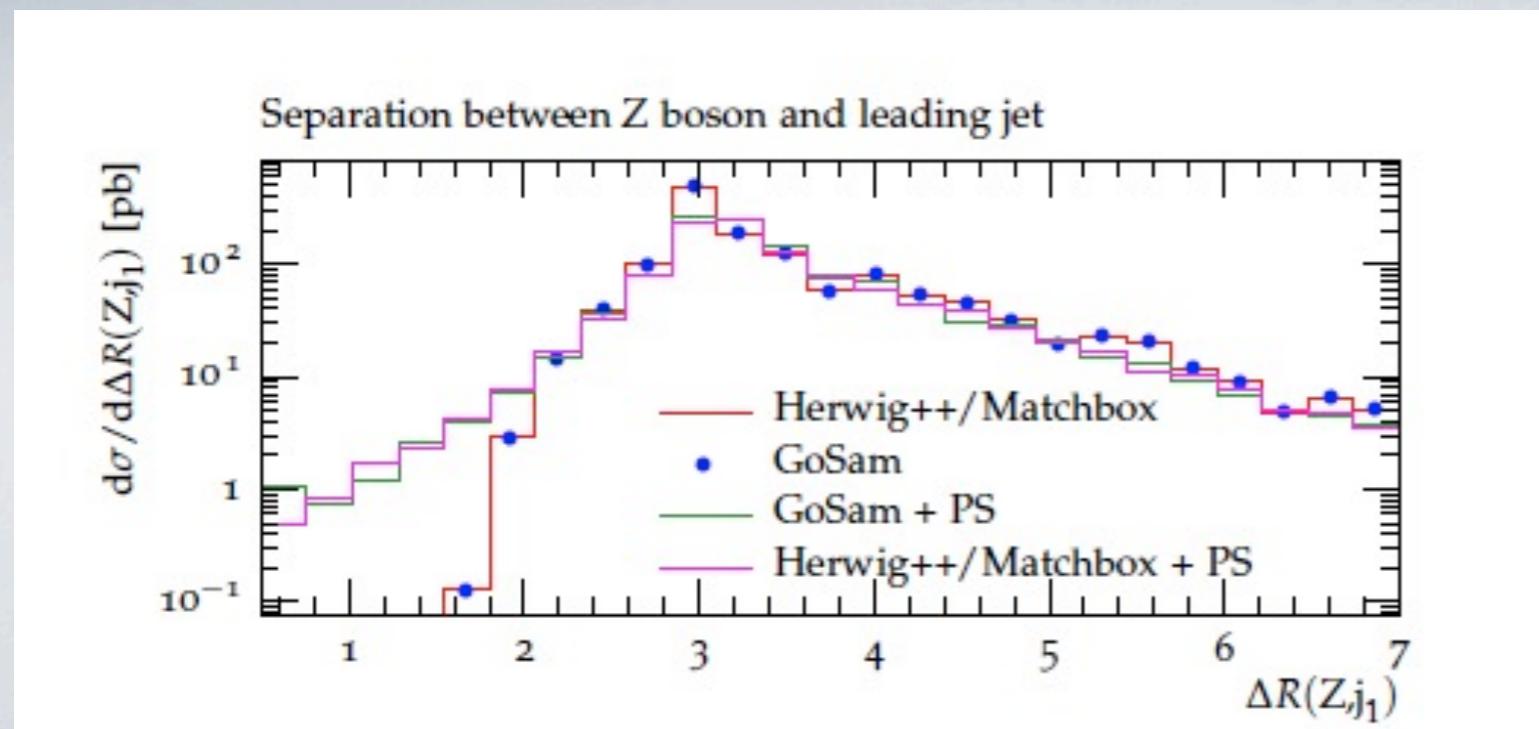
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- colour- and spin-correlated tree amplitudes  
can be used e.g. to **build subtraction terms** for NLO real radiation



[Bellm, Gieseke, Greiner, GH, Plätzer,  
Reuschle, von Soden-Fraunhofen '13]

# BSM applications of GoSam

$pp \rightarrow (\text{graviton} \rightarrow \gamma\gamma) + 1 \text{ jet}$  [Greiner, GH, Reichel, von Soden-Fraunhofen '13]

within ADD models of large extra dimensions

non-standard propagator for gravitons  $\Rightarrow$  `customspin2prop` in GoSam

involves up to rank 5 box integrals, complicated tensor structure

import of **model file** in **UFO** (Universal FeynRules Output [Degrade, Duhr et al.] ) format

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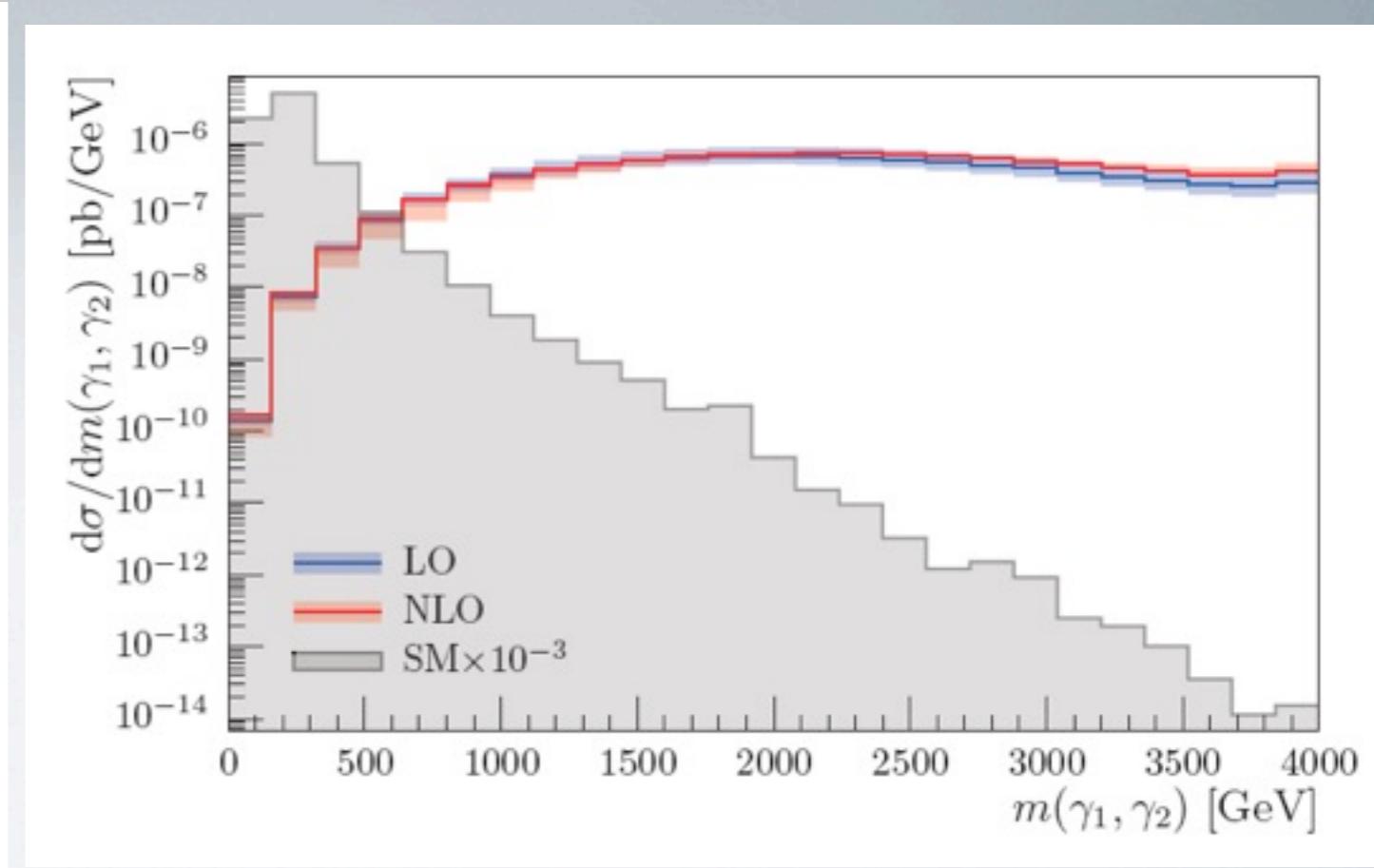
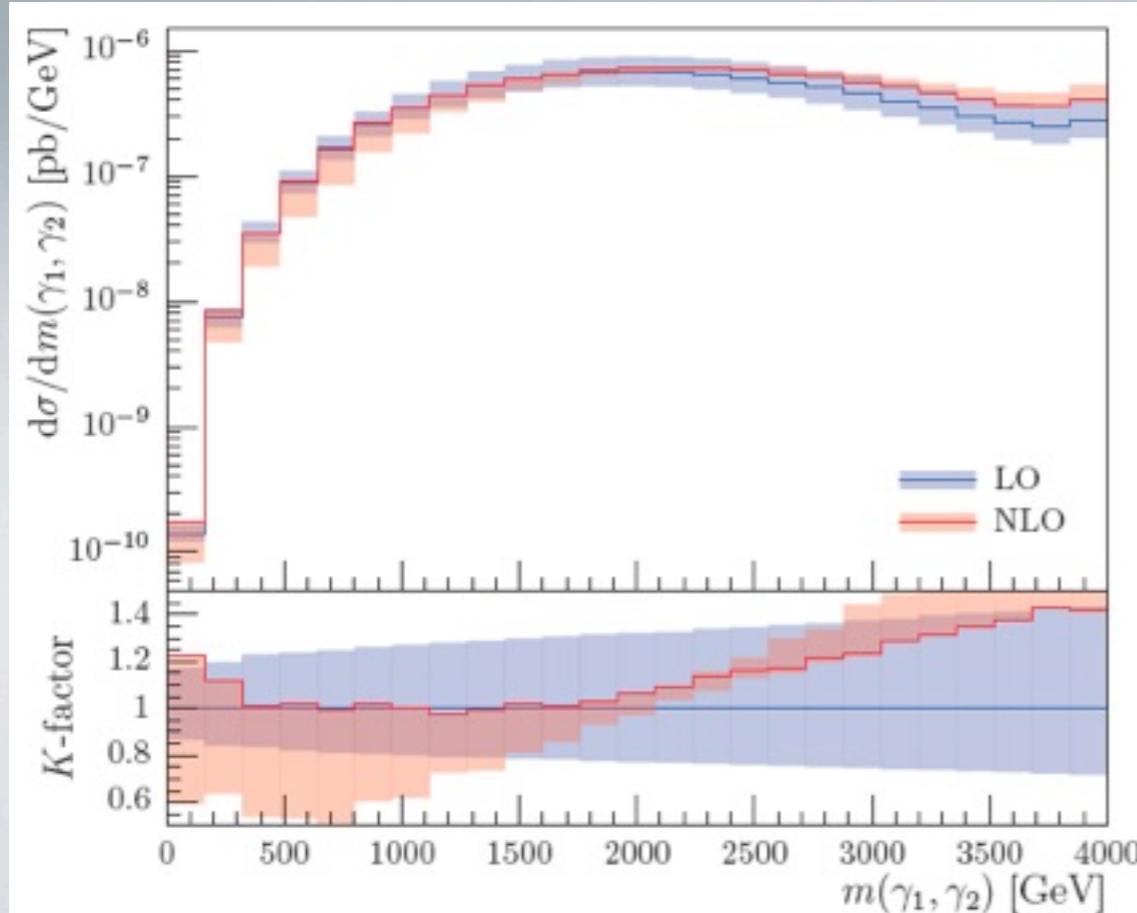
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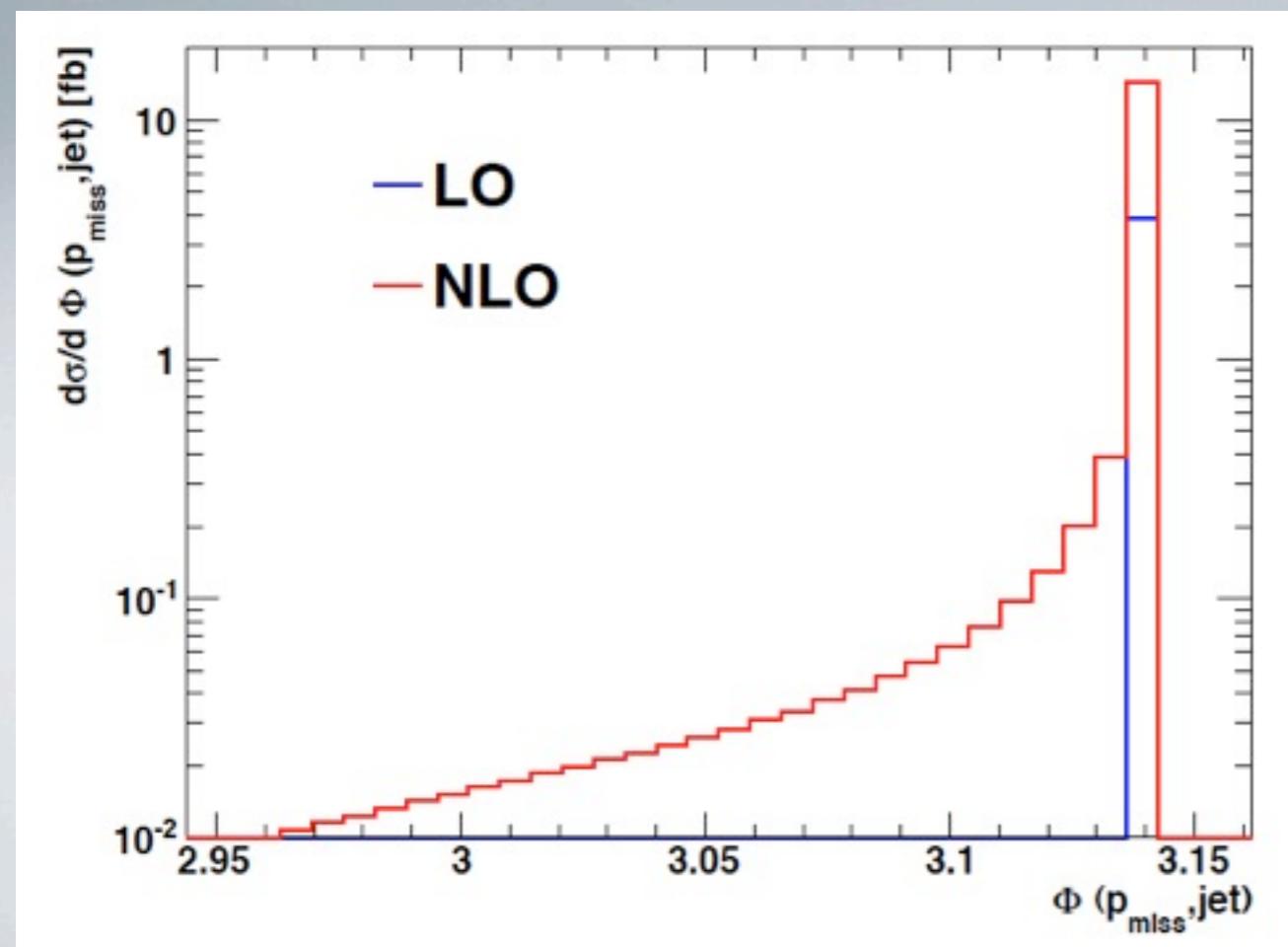
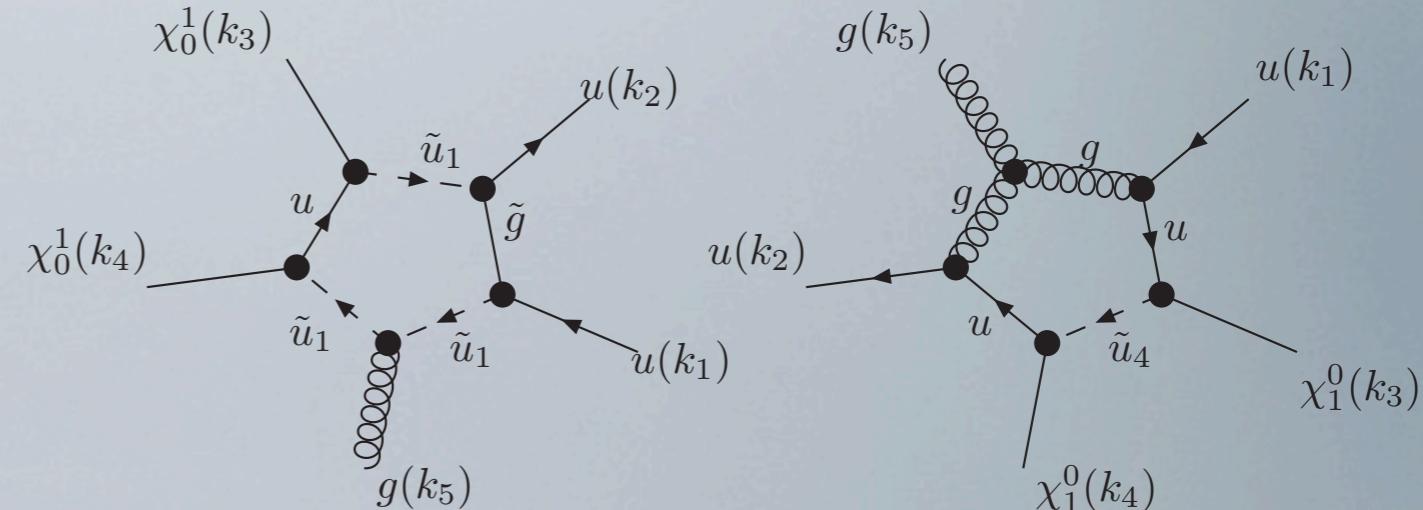
# BSM applications of GoSam

$pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 + jet$   
 (SUSY QCD corrections)

[Cullen, Greiner, GH '13]

signature: monojet + missing ET

- full off-shell effects included
- complex masses
- complicated phase space structure
- UFO model file import,  
renormalisation done separately



SUSY Parameters	
$M_{\tilde{\chi}_1^0} = 299.5$	$\Gamma_{\tilde{\chi}_1^0} = 0$
$M_{\tilde{g}} = 415.9$	$\Gamma_{\tilde{g}} = 4.801$
$M_{\tilde{u}_L} = 339.8$	$\Gamma_{\tilde{u}_L} = 0.002562$
$M_{\tilde{u}_R} = 396.1$	$\Gamma_{\tilde{u}_R} = 0.1696$
$M_{\tilde{d}_L} = 348.3$	$\Gamma_{\tilde{d}_L} = 0.003556$
$M_{\tilde{d}_R} = 392.5$	$\Gamma_{\tilde{d}_R} = 0.04004$
$M_{\tilde{b}_L} = 2518.0$	$\Gamma_{\tilde{b}_L} = 158.1$
$M_{\tilde{b}_R} = 2541.8$	$\Gamma_{\tilde{b}_R} = 161.0$
$M_{\tilde{t}_L} = 2403.7$	$\Gamma_{\tilde{t}_L} = 148.5$
$M_{\tilde{t}_R} = 2668.6$	$\Gamma_{\tilde{t}_R} = 182.9$

angle between leading jet and  
missing momentum

# SM applications of GoSam

$$pp \rightarrow W^+W^- b\bar{b} \quad (m_b = 0)$$

GoSam + Sherpa

[GH, Maier, Nisius, Schlenk, Winter '13]

[Denner, Dittmaier, Kallweit, Pozzorini '11]

[Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek '11]

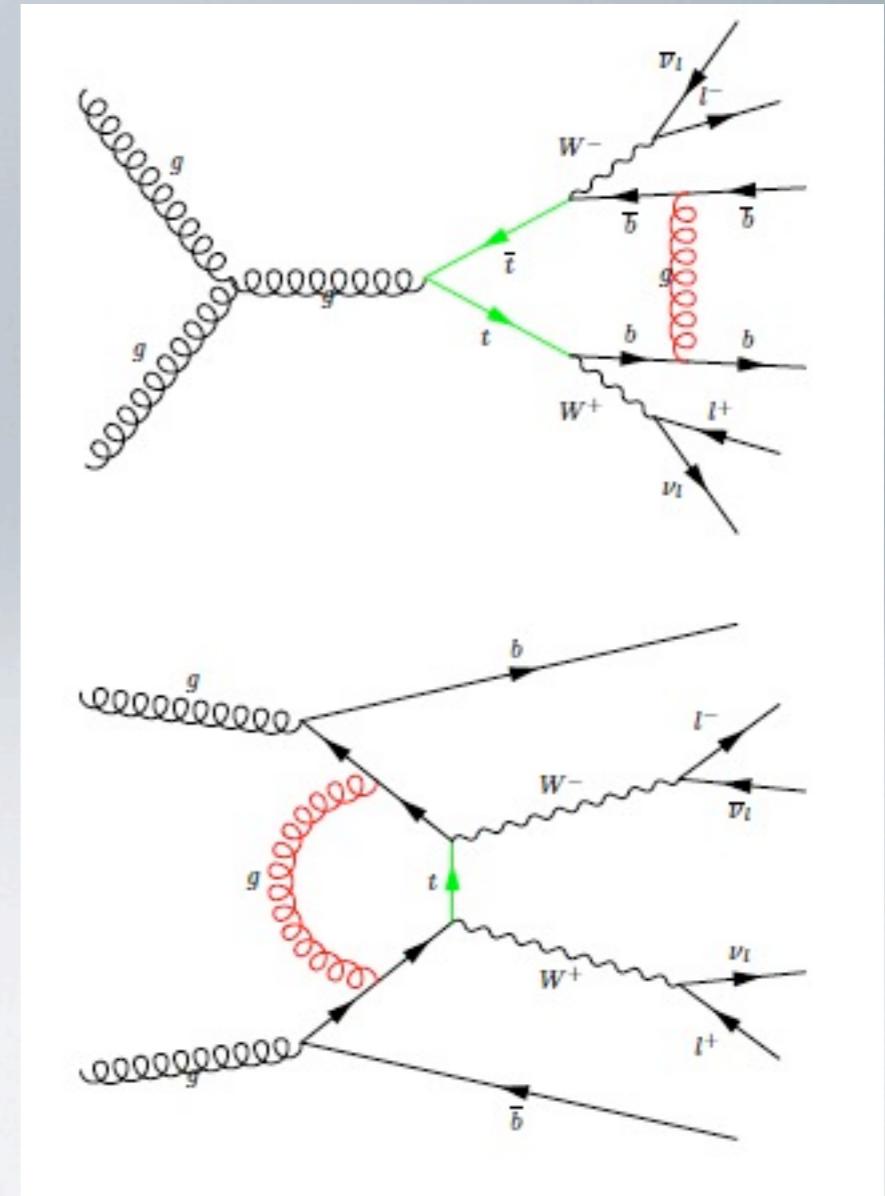
investigate influence of non-factorizing  
and non-resonant contributions on

**top mass determination**

- leptonic W-decays (resonant)
- use  $m_{lb}^2 = (p_{b\text{-jet}} + p_l)^2$  for mass

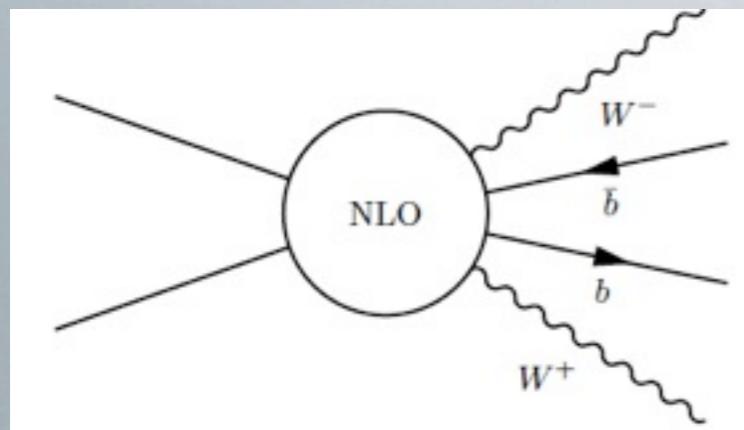
measurement, following [ATLAS-CONF-2013-77](#)  
(template method)

- analysis is sensitive to the **shape** of the distribution (normalized)

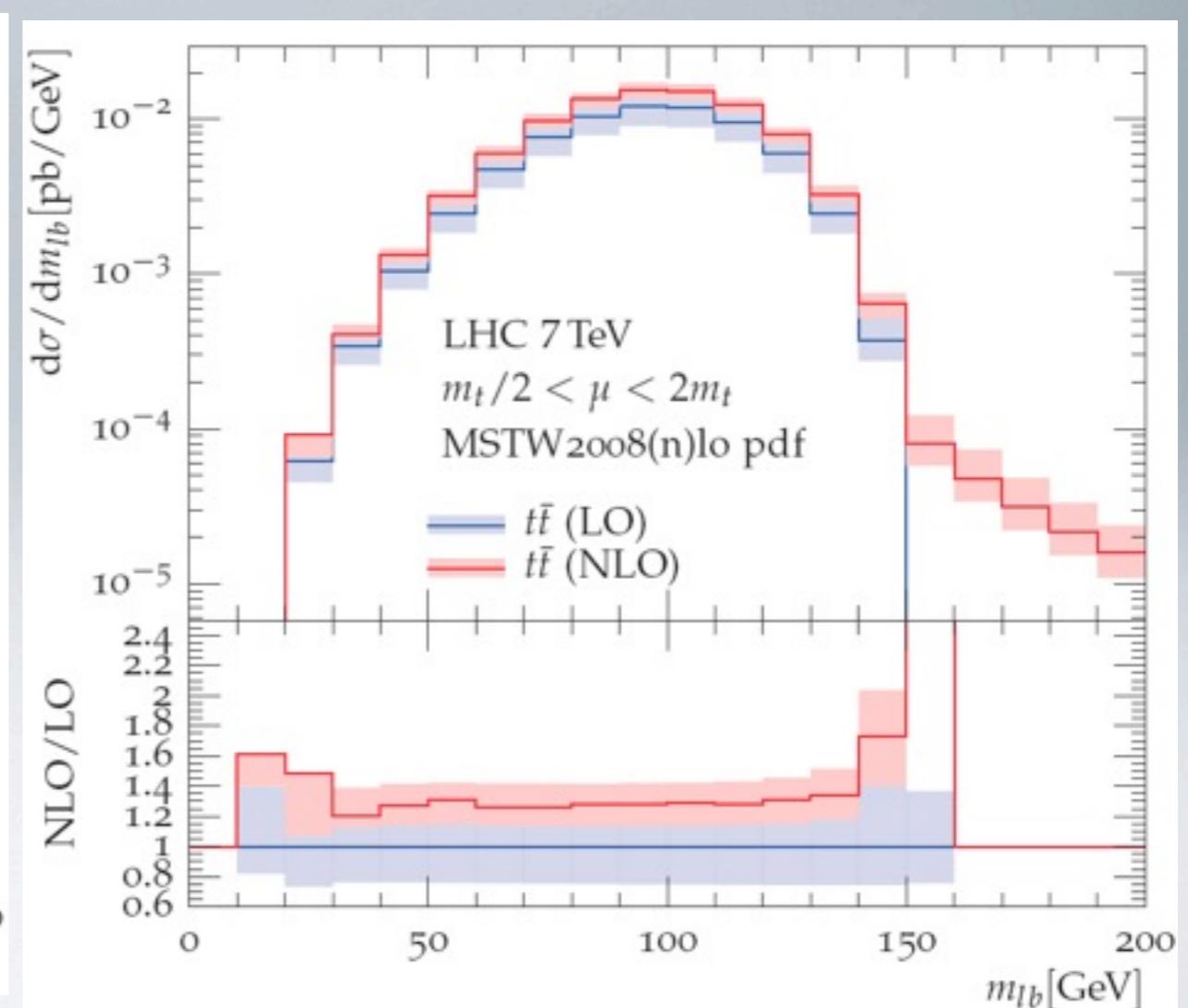
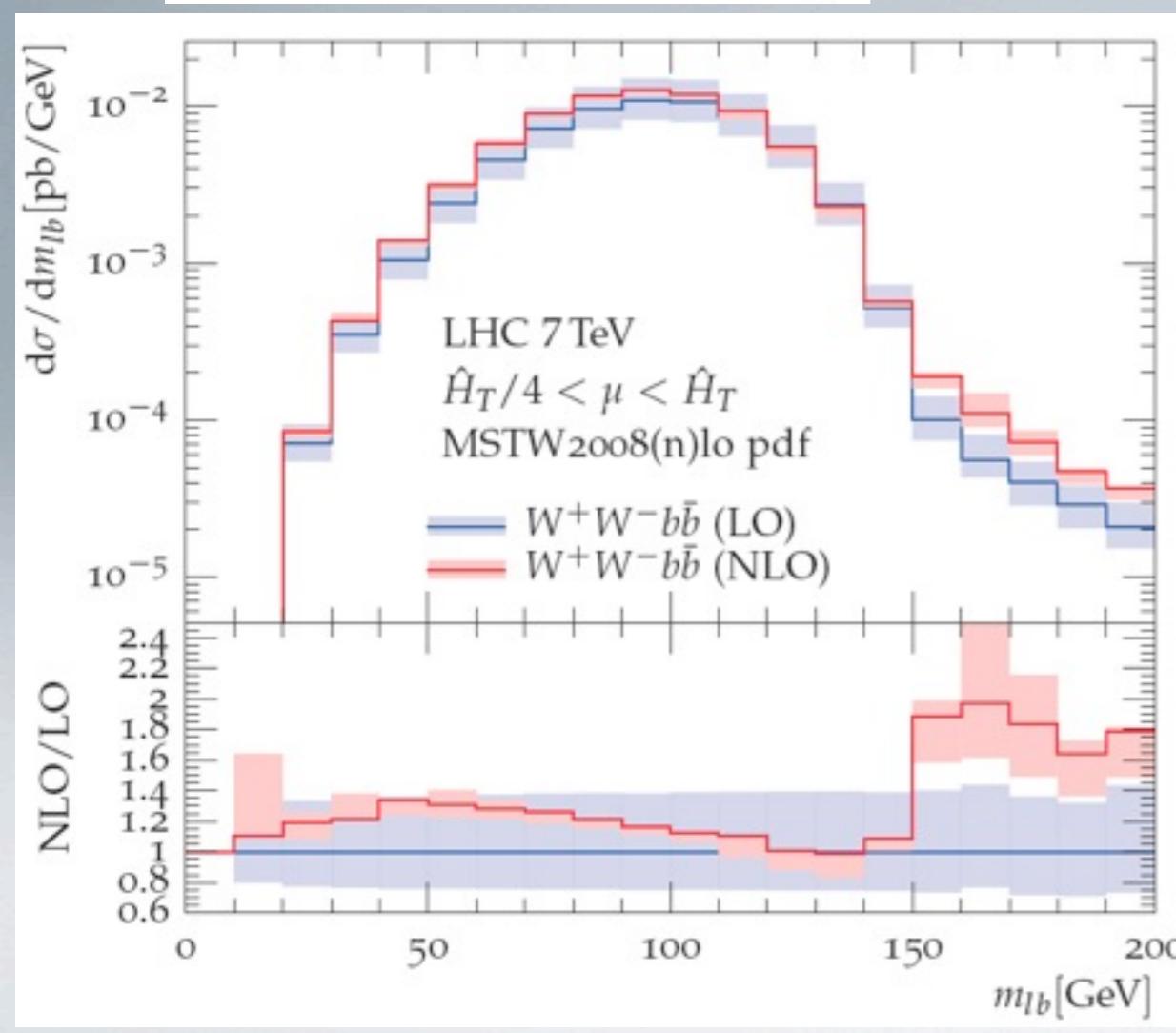
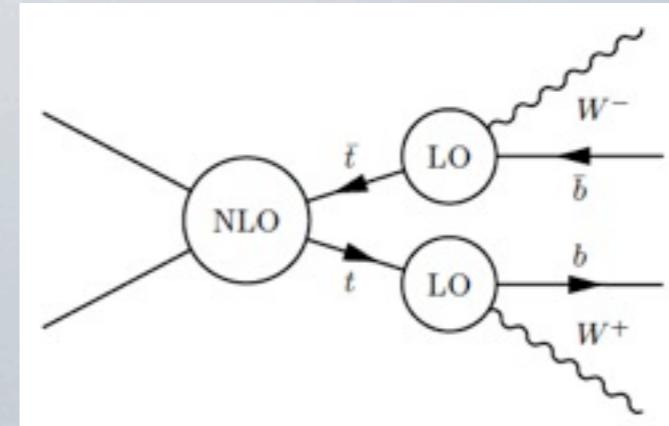


- compare full versus factorized calculation for observable  $m_{lb}$

full (WWbb)

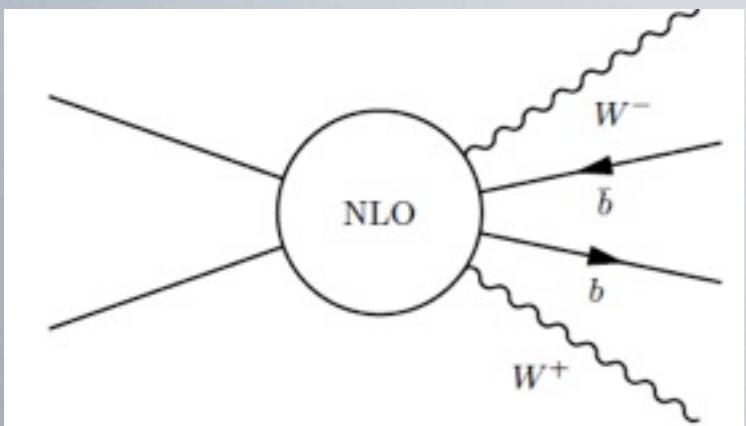


factorized ( $t\bar{t}$ )

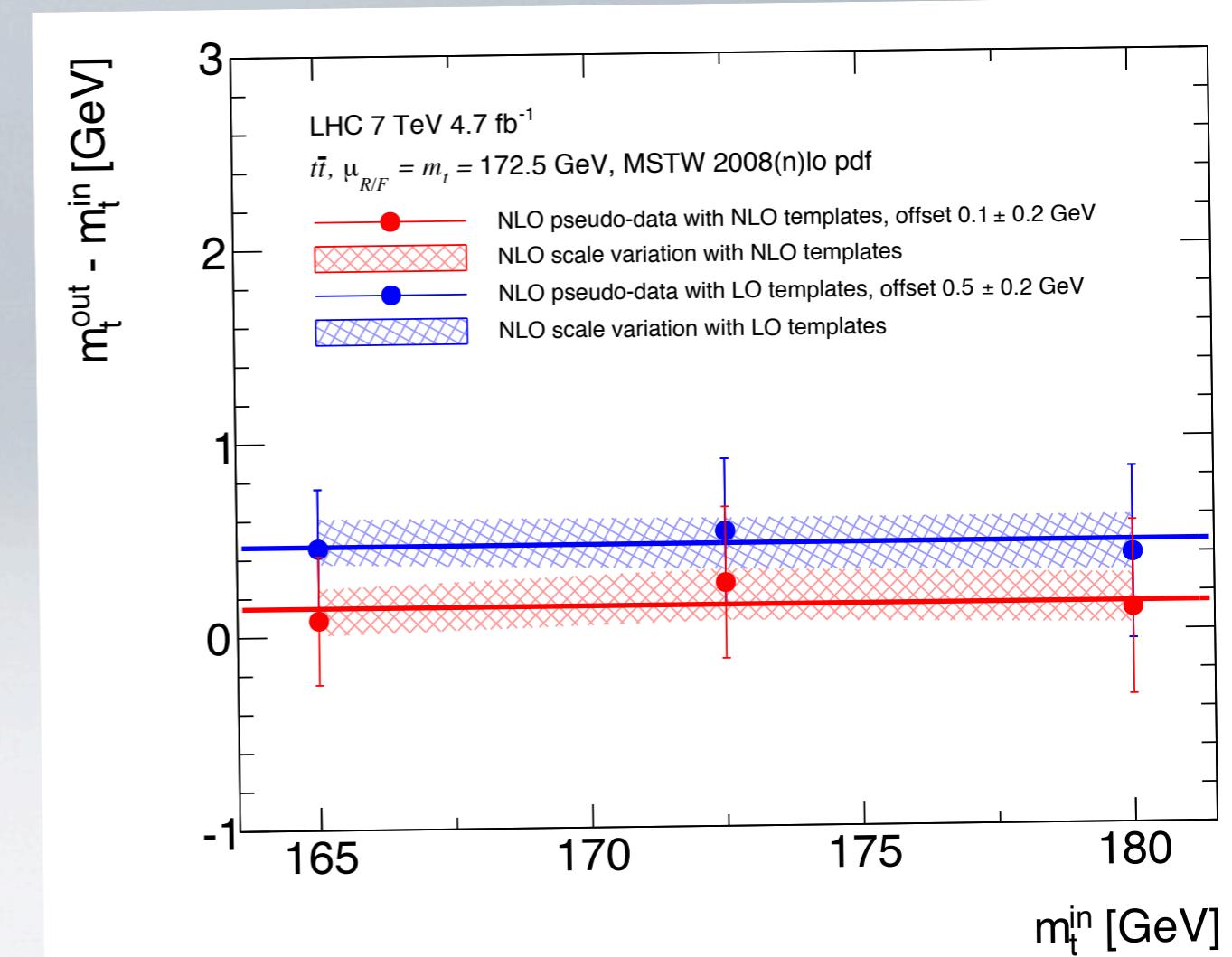
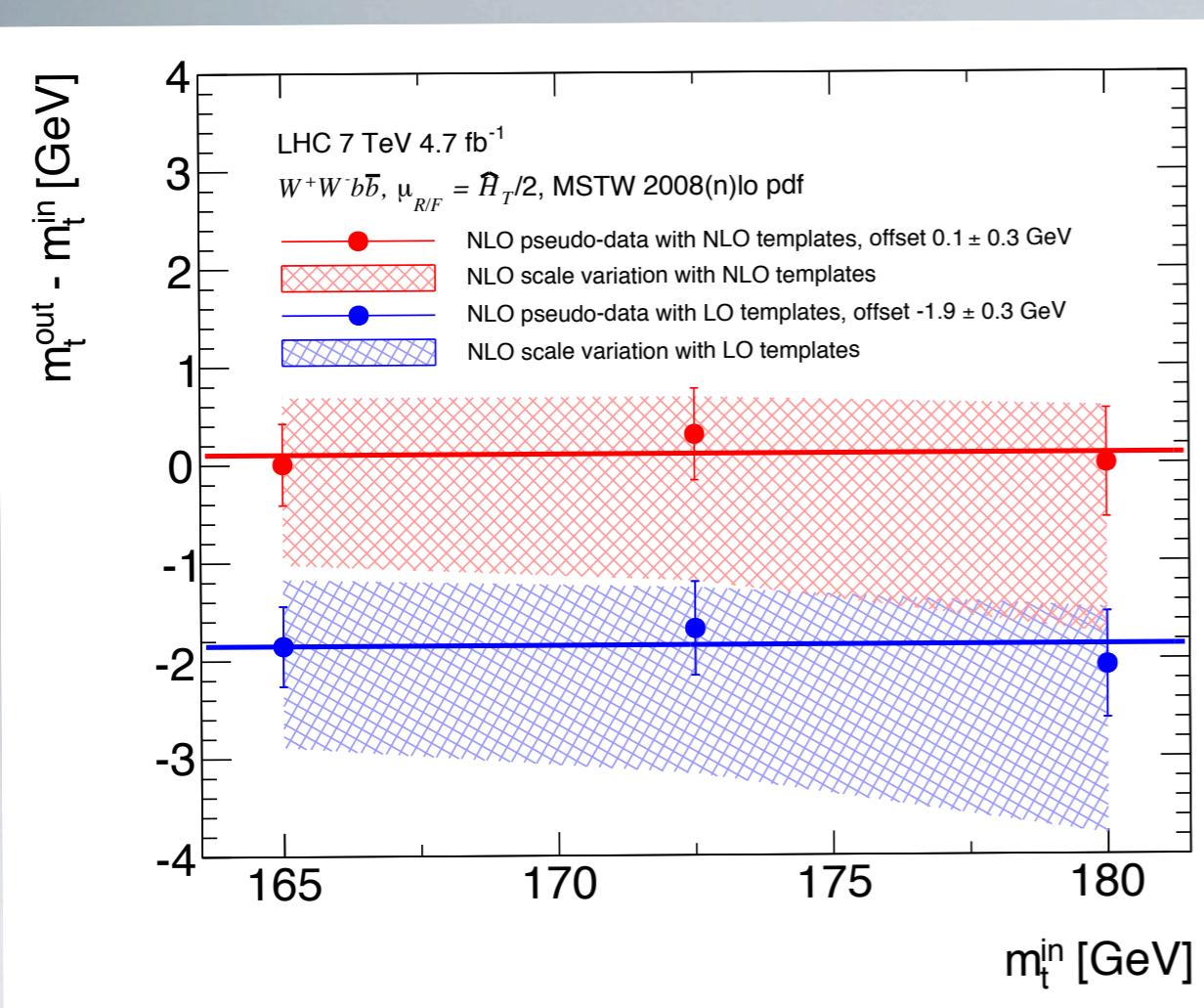
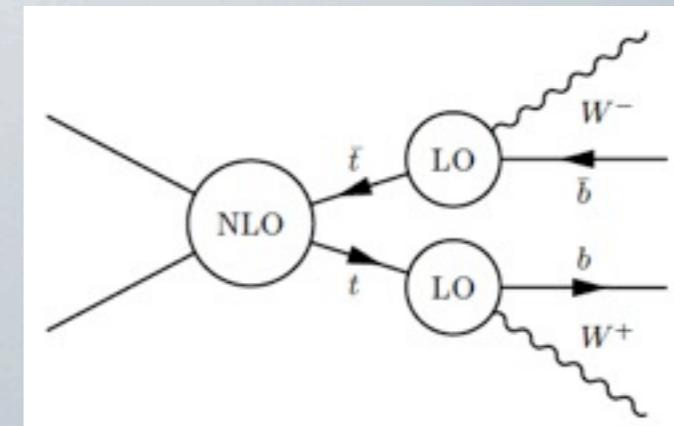


shape differences in full calculation, amplified by scale variations,  
have important consequences on uncertainties on mttop

- compare full versus factorized calculation for observable  $m_{lb}$
- full (WWbb)



factorized ( $t\bar{t}$ )



- uncertainties from scale variations larger in full approach:  $^{+0.6}_{-1.0} \text{ GeV}$  (full) vs.  $\pm 0.2 \text{ GeV}$  (factorized)
- shift between NLO / LO template fit:  $\sim 1.9 \text{ GeV}$  (full) vs.  $\sim 0.5$  (factorized)

# Installation and usage of GoSam

**installation:** installation script downloads GoSam and reduction libraries and installs everything

```
 wget http://gosam.hepforge.org/gosam-installer/gosam\_installer.py
```

```
 chmod +x gosam_installer.py
```

```
 ./gosam_installer.py [ --prefix=installation_path ]
```

installation script will also install FORM [J.Vermaseren et al.]  
and QGraf [P. Nogueira] if not present already

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**usage:** create template for input file process.in:

gosam.py --template process.in

edit input file process.in

to generate amplitude (standalone):

gosam.py process.in

within BLHA:

gosam.py --olp order.lh

example input file:

```
process_name=eett
process_path=eett
in=    e+, e-
out=   t, t~
model= smdiag
model.options=ewchoose
order= gs, 0, 2
zero=me
one=gs,e
regularisation_scheme=dred
```

many more options available, will take defaults if not set

# Summary

new version **GoSam-2.0**:  
efficient, multi-purpose, automated tool for  
one-loop multi-leg calculations

- more compact code, faster evaluation times
  - large range of applicability: QCD, electroweak, BSM  
(higher rank integrals, complex masses, model file import)
  - new reduction method (library **Ninja**)
  - refined stability tests and rescue systems
  - large flexibility for combination with Monte Carlo programs
  - can also produce spin-and colour correlated tree amplitudes
- ⇒ provides all building blocks for NLO real radiation
- easy installation and usage



# Summary

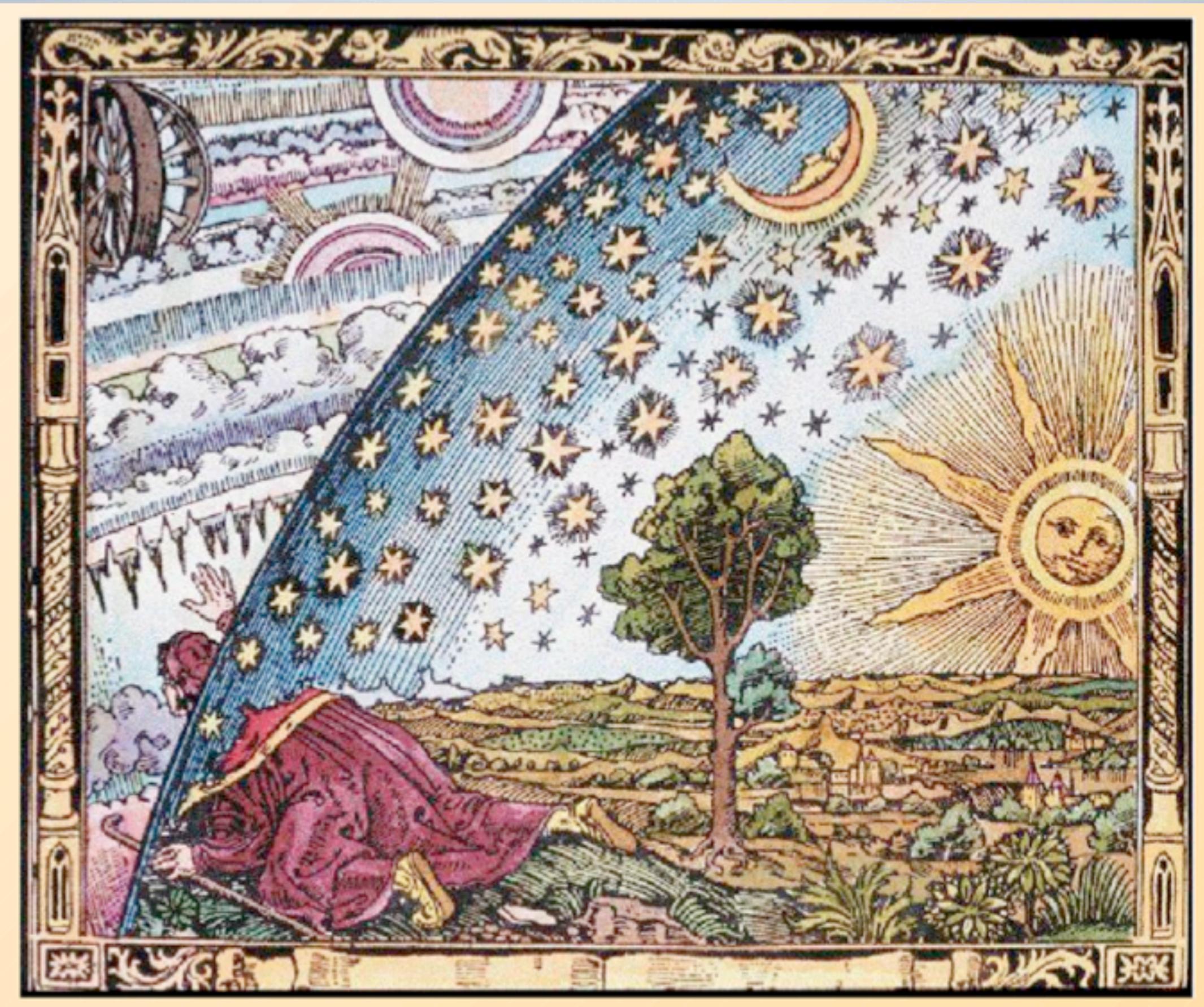
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one-loop multi-leg calculations

- more compact code, faster evaluation times
- large range of applicability: QCD, electroweak, BSM  
(higher rank integrals, complex masses, model file import)
- new reduction method (library **Ninja**)
- refined stability tests and rescue systems
- large flexibility for combination with Monte Carlo programs
- can also produce spin-and colour correlated tree amplitudes
  - ⇒ provides all building blocks for NLO real radiation
- easy installation and usage

looking forward to a multitude of  
phenomenological applications !



# Additional Slides



# GoSam input card options

```
1 process_name=eett
2 process_path=eett
3 in=    e+, e-
4 out=   t, t~
5 model= smdiag
6 model.options=ewchoose
7 order= gs, 0, 2
8 zero=me
9 one=gs,e
10 regularisation_scheme=dred
11 helicities=
12 qgraf.options=onshell,notadpole,nosnail
13 qgraf.verbatim= True=iprop[Z, 0, 0];\n\
14           true=iprop[H, 0, 0];
15 qgraf.verbatim.lo=
16 qgraf.verbatim.nlo=
17 polvec=numerical
18 diagsum=True
19 reduction_programs=ninja,golem95,samurai
20 extensions=shared
21 debug=nlo
22 select.lo=
23 select.nlo=
24 filter.lo=
25 filter.nlo=
26 filter.module=
27 renorm_beta=True
28 renorm_mqwf=True
29 renorm_decoupling=True
30 renorm_mqse=True
31 renorm_logs=True
32 renorm_gamma5=True
33 reduction_interoperation=-1
34 reduction_interoperation_rescue=-1
35 samurai_scalar=2
36 nlo_prefactors=0
37 PSP_check=True
38 PSP_rescue=True
39 PSP_verbosity=False
40 PSP_chk_th1=8
41 PSP_chk_th2=3
42 PSP_chk_th3=5
43 PSP_chk_kfactor=10000
44 reference-vectors=
45 abbrev.limit=0
```

```
46 templates=
47 qgraf.bin=qgraf
48 form.bin=form
49 form.threads=2
50 form.tempdir=/tmp
51 haggies.bin=
52 fc.bin=/usr/bin/gfortran
53 python.bin=python
54 ninja.fcflags=
55 ninja.ldflags=
56 samurai.fcflags=
57 samurai.ldflags=
58 golem95.fcflags=
59 golem95.ldflags=
60 r2=explicit
61 symmetries=family,generation
62 crossings=
```

# stability tests and rescue system

- pole test :

$$\delta_{pole} = \left| \frac{S_{IR} - S}{S_{IR}} \right|$$

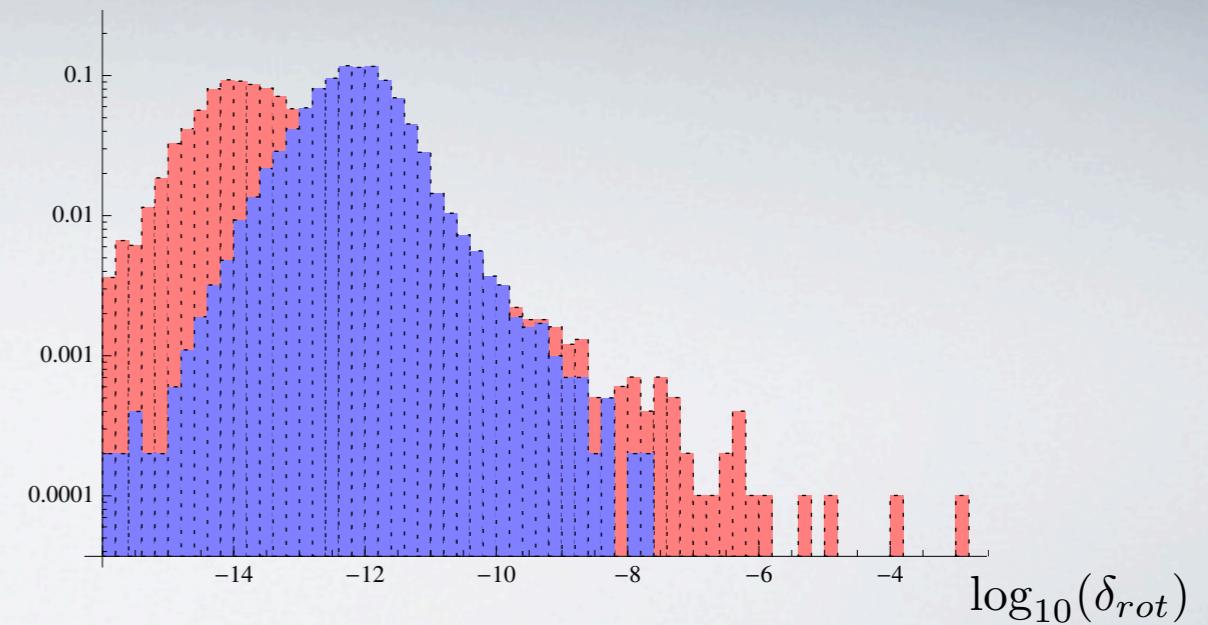
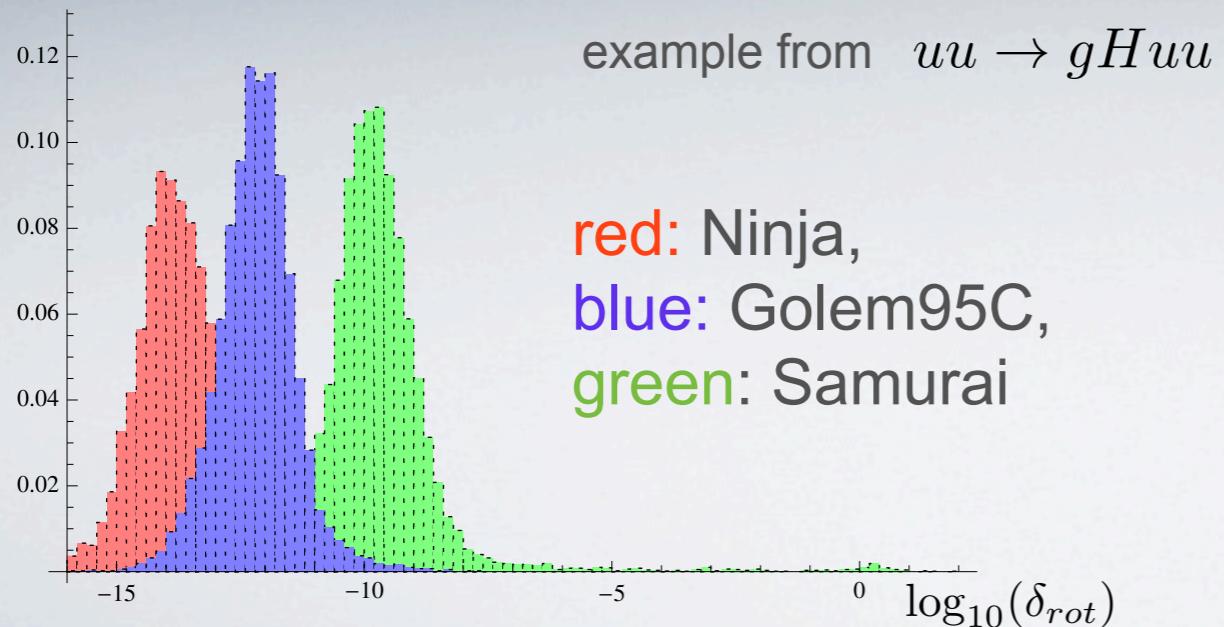
$$P_{pole} = -\log_{10}(\delta_{pole})$$

- rotation test:

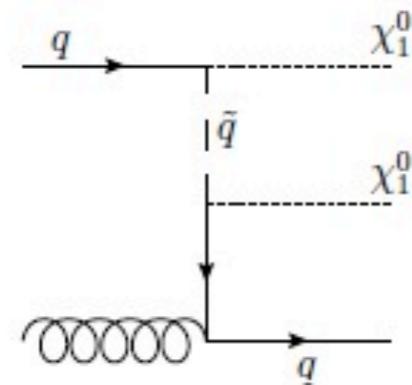
$$\delta_{rot} = 2 \left| \frac{A_{rot}^{\text{fin}} - A^{\text{fin}}}{A_{rot}^{\text{fin}} + A^{\text{fin}}} \right|$$

- three thresholds  $P_{\text{high}}$  (default 8),  $P_{\text{low}}$  (default 3),  $P_{\text{set}}$  (default 5)

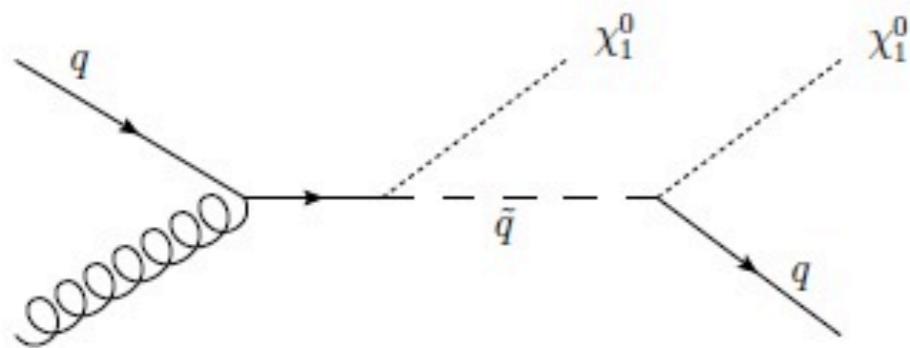
- if  $P_{\text{pole}} > P_{\text{high}}$ : accept
- if  $P_{\text{pole}} < P_{\text{low}}$ : discard
- if  $P_{\text{high}} > P_{\text{pole}} > P_{\text{low}}$ : do rotation test, discard if  $P_{\text{rot}} < P_{\text{set}}$



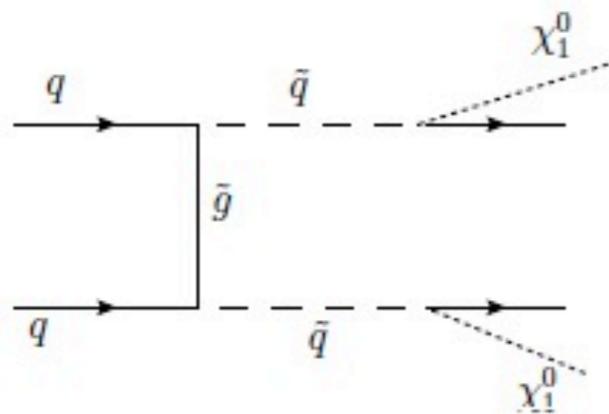
- GoSam default: reduction with Ninja, rescue with golem95C



t-channel squark exchange



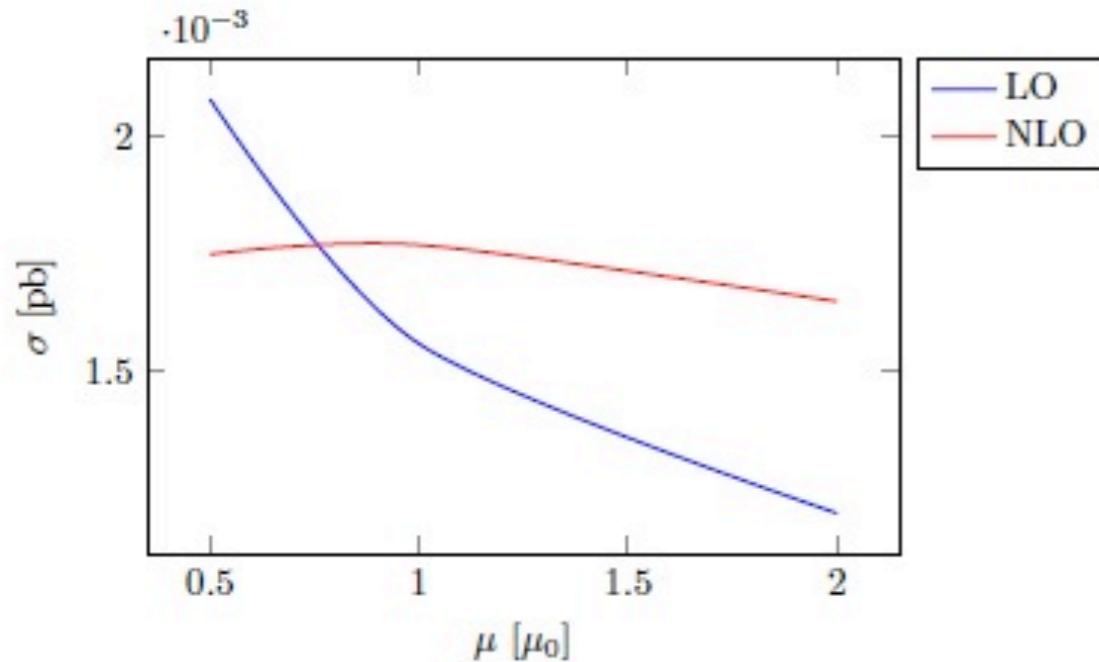
s-channel squark exchange



appears at NLO, can also be  
regarded as LO for squark pair  
production  
 $\Rightarrow$  huge contribution

$pp \rightarrow (\text{graviton} \rightarrow \gamma\gamma) + 1 \text{ jet}$

	cross section [fb]	MC error [fb]	scale uncertainty [fb]	
LO	1.561	$\pm 6.5 \times 10^{-4}$	0.522 -0.363	$\mu = \mu_0/2$ $\mu = 2\mu_0$
NLO	1.767	$\pm 7.1 \times 10^{-3}$	-0.02 -0.11	$\mu = \mu_0/2$ $\mu = 2\mu_0$



### Cuts and parameters

$$p_{T,\gamma} \geq 25 \text{ GeV} \quad |\eta_\gamma| \leq 2.5 \quad 0.4 \leq \Delta R_{\gamma\gamma}$$

$$140 \text{ GeV} \leq m_{\gamma\gamma} < 3.99 \text{ TeV}$$

$$p_{T,\text{leading jet}} \geq 30 \text{ GeV} \quad |\eta_{\text{jet}}| \leq 4 \quad 0.4 \leq \Delta R_{\text{jet},\gamma}$$

$$\mu_0^2 = \mu_F^2 = \frac{1}{4} (m_{\gamma\gamma}^2 + p_{T,jet}^2)$$

4 (5 u. 6) extra dimensionens  $M_s = 4 \text{ TeV}$